Computing resolutions of equivariant modules

Sebastian Posur

University of Siegen

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Equivariant resolutions

In this computation presentation, we will compute equivariant resolutions of $\mathbb{Z}$-graded modules over the exterior algebra in a **categorical** way.

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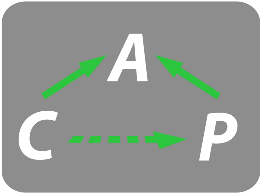

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- CAP serves as a categorical programming language with categorical operations as primitives.
- CAP simplifies the implementation of specific instances of categories

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We call $\bigoplus_{i \in \mathrm{Irr}(G)} k\text{-vec}$ a **skeletal model** for $\mathrm{Rep}_k(G)$.

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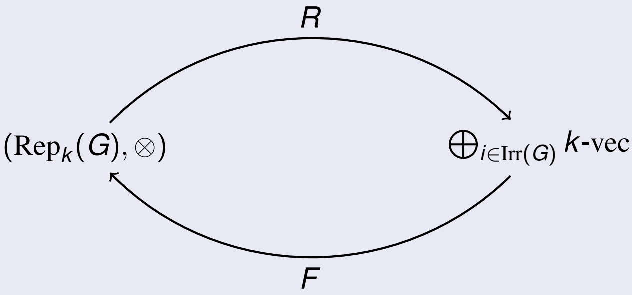
What about tensor products?

Structure transport



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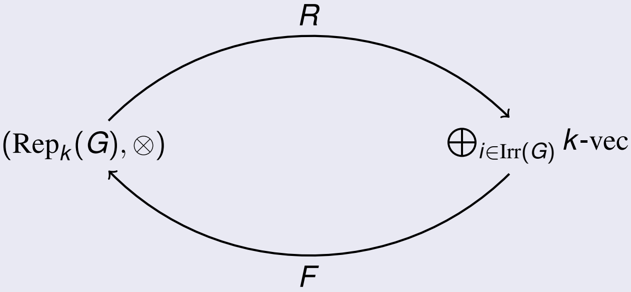

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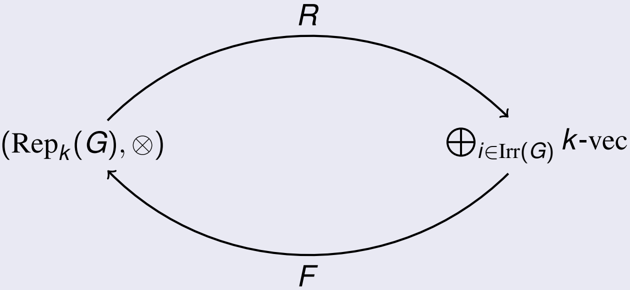

$$\otimes_{\text{skeletal}} := R \circ \otimes \circ (F \times F)$$

Also works for associators $A \otimes (B \otimes C) \to (A \otimes B) \otimes C$

Internal algebra

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\end{array}
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Free modules

$$\begin{array}{c} (B \otimes E) \otimes E \\ \wr \big\downarrow \text{AssociatorLeftToRight}(B, E, E) \\ B \otimes (E \otimes E) \\ \big\downarrow B \otimes \mu_E \\ B \otimes E \end{array}$$

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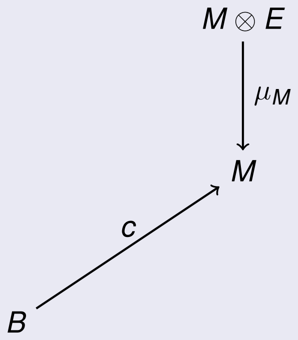
Universal property of free modules



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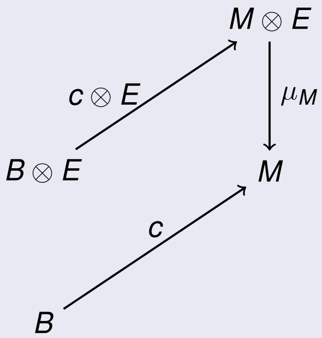

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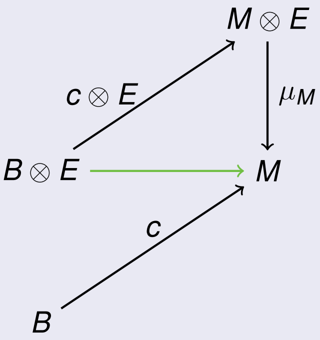

Equivariant cohomology table

H^2 :	$\chi_4 + 2\chi_2$	$\chi_1 + \chi_3$	.	.	.	.	.	.
H^1 :	.	.	χ_1	χ_2	χ_4	χ_5	.	.
H^0 :	.	.	.	.	.	.	$\chi_3 + \chi_5$	$2\chi_4 + \chi_2$
p	-4	-3	-2	-1	0	1	2	3

The equivariant cohomology table of a rank 2 vector bundle.

Download CAP

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http://homalg-project.github.io/CAP_project/