Nice, June 12 - 16, 2017

INTERNATIONAL CONFERENCE ON EFFECTIVE METHODS IN ALGEBRAIC GEOMETRY

Effective
Effektive
Эффективные
Effettivi
Effectives
Efectivos

Methods
Methodem
Методы
Metodi
Méthodes
Métodos

M E G A

in Algebraic
in Algebraischer
в Алгебраической
Algebrica
Algébrique
Algebraica

Geometry
Geometrie
Геометрии
In Geometria
en Géométrie
en Geometria

Book of abstracts

An event organized by

Inria informatics mathematics

Université Nice Sophia Antipolis
Laboratoire J.-A. Dieudonné

CNRS

Member of UNIVERSITÉ CÔTE D'AZUR

MEGA 2017

Contents

1 Invited talks

Automorphisms of P^1 bundles over rational surfaces

Jérémy Blanc

In this talk, I will present a way to compute automorphisms of P^1-bundles over rational surfaces. As it will be explained, we can reduce to the case of Hirzebruch surfaces, and then study the action on the fibres in an explicit way, using transition functions.

Joint work with Andrea Fanelli and Ronan Terpereau.

Constructions of k-regular maps using finite local schemes

Jarosław Buczyński

A continuous map $\mathbb{R}^m \to \mathbb{R}^N$ or $\mathbb{C}^m \to \mathbb{C}^N$ is called k-regular if the images of any k distinct points are linearly independent. Given integers m and k a problem going back to Chebyshev and Borsuk is to determine the minimal value of N for which such maps exist. The methods of algebraic topology provide lower bounds for N, however there are very few results on the existence of such maps for particular values m. During the talk, using the methods of algebraic geometry, we will construct k-regular maps. We will relate the upper bounds on the minimal value of N with the dimension of the a Hilbert scheme. The computation of the dimension of this space is explicit for $k \leq 9$, and we provide explicit examples for k at most 5. We will also provide upper bounds for arbitrary m and k. The problem has its interpretation in terms of interpolation theory: for a topological space X and a vector space V, a map $X \to V$ is k-regular if and only if the dual space V^* embedded in space of continuous maps from X to the base field $\mathbb{R}$ or $\mathbb{C}$ is k-interpolating, i.e. for any k distinct points $x_1, \ldots, x_k$ of X and any values f_i, there is a function in V^*, which takes values f_i at x_i. Similarly, we can interpolate vector valued continuous functions, and analogous methods provide interesting results.

The talk is based mainly on:

Constructions of k-regular maps using finite local schemes, Jarosław Buczyński, Tadeusz Januszkiewicz, Joachim Jelisiejew, Mateusz Michałek, to appear in Journal of European Mathematical Society, arXiv:1511.05707

Examples of k-regular maps and interpolation spaces, Mateusz Michałek, Christopher Miller, arXiv:1512.00609

And follow up work in progress with Tadeusz Januszkiewicz.

Fundamental operations on rank metric codes

Eimear Byrne

We outline recent progress in rank-metric coding theory, making reference to the classical theory of Hamming metric codes. In the history of Hamming metric codes, fundamental coding theoretic operations such as puncturing and shortening have played an important role in questions of code optimality and code constructions. They have also arisen in the literature on the zeta function of a Hamming metric code (Duursma 2001).

Rank-metric codes have featured prominently in the literature on algebraic codes in recent years. They have applications to error-correction in multicast networks and have been considered in code-based cryptosystems. Such codes are subsets of the matrix ring $\mathbb{F}_q^{m\times n}$ endowed with the rank distance function, which measures the $\mathbb{F}_q$-rank of the difference of a pair of matrices. A small number of families of optimal rank metric codes have been known for some decades (Delsarte 1978, Gabidulin 1985, Roth 1991). These are examples of maximum rank distance (MRD) codes. Very recently, more general families of MRD codes have been discovered (Sheekey 2016) and there has been much activity in research into such codes. However, the general theory of rank metric codes is still very much open.

We describe properties of shortened and punctured codes for matrix codes, with respect to the rank metric. We introduce a number of ways to define such operations and give a duality result. We use the notion of a rank metric shortened code to describe the rank-metric zeta function of a code in terms of a generating function and relate this object to the rank-metric weight enumerator of a code. We show that, as in the Hamming metric case, the normalized weight enumerator of a rank-metric code is invariant under shortening and puncturing. We outline the differences between the Hamming and rank metric theories on this topic. We discuss the rank-metric covering problem and introduce bounds on the rank metric covering radius. We show how these results can be applied to the classification of MRD and quasi-MRD codes. We describe several open problems in the theory of rank-metric codes.

Virtual refinements of the Vafa-Witten formula

Lothar Goettsche

Based of physics arguments Vafa and Witten made predictions about the Euler numbers of moduli spaces of sheaves on surfaces. They give explicit generating functions in terms of modular forms. These moduli spaces are in general very singular, but they have a perfect obstruction theory, and thus a virtual fundamental class and a virtual Tangent bundle, and thus virtual Chern numbers and in particular a virtual Euler number. In a virtual sense they are smooth projective varieties and carry the virtual versions of the invariants of smooth projective varieties.

We interpret the Vafa-Witten prediction as being about the virtual Euler numbers. Then a formula of Mochizuki allows to compute the virtual Euler numbers in terms of integrals on Hilbert schemes of points, which we carry out via reduction to toric surfaces and applying Atiyah-Bott localization. These computations are carried out with a suitable program. This allows to check the conjecture in a wide variety of cases up to high expected dimensions of the moduli spaces. Further computations allow us to extend the conjectures first to the χ_y-genus and then to the elliptic genus, where we obtain generating functions similar to that of Dijkgraaf-Moore-Verlinde-Verlinde for Hilbert schemes of points. Finally we extend the conjectures to the virtual cobordism class of the moduli spaces, which encodes all its virtual Chern numbers.

This is joint work with Martijn Kool

Injectivity in Phase retrieval

Milena Hering

In signal processing, often one cannot measure a complex vector directly, but instead one can measure the modulus of its inner product with a given spanning set. We use algebraic geometry to study the question under what conditions on this spanning set the original vector can be uniquely obtained up to a global phase factor.

This is joint work with Aldo Conca, Dan Edidin, and Cynthia Vinzant.

Negative correlation and Hodge-Riemann relations

June Huh

All finite graphs satisfy the two properties mentioned in the title. I will explain what I mean by this, and speculate on generalizations and interconnections. This talk will be non-technical: Nothing will be assumed beyond basic linear algebra.

Periods in action

Pierre Lairez

The study of integrals of algebraic functions has tremendous benefits in algebraic geometry, an early example is Euler's work on elliptic integrals. Lesser known are the applications to computational mathematics. I will expose some of them.

Specifically, the talk will focus on periods, that is the result of integrating a multivariate rational function on a cycle. I will show how to compute them, in the form of differential equations, and I will describe two applications to very different problems: the automated proof of binomial identities and the computation of the volume of certain semi-algebraic sets.

Macaulay style formulae for the sparse resultant

Martin Sombra

The sparse resultant is a classical object from elimination theory, that has been widely used in polynomial equation solving and that has strong connections with combinatorics, toric geometry, residue theory, and hypergeometric functions.

In this talk, I will review some of the matrix formulae for this object. The first ones go back to Cayley and Sylvester in the univariate case, and to Macaulay in the dense multivariate case. Formulae for the sparse case were obtained by Canny-Emiris and by D'Andrea, and simplified in a recent work in collaboration with D'Andrea and Jeronimo.

2 Selected talks

The tropical analogue of the Helton-Nie conjecture is true

Xavier Allamigeon, Stephane Gaubert, Mateusz Skomra

Helton and Nie conjectured that every convex semialgebraic set over the field of real numbers can be written as the projection of a spectrahedron. We show that the following consequence of this conjecture is true: over a real closed nonarchimedean field of Puiseux series, the convex sets that are R-definable and the projections of spectrahedra have precisely the same images by the nonarchimedean valuation. In other words, the tropical analogue of the Helton-Nie conjecture is true. The proof relies on game theory methods.

Solving parameterized polynomial systems with decomposable projections

Carlos Améndola Jose Israel Rodriguez

The Galois/monodromy group of a parameterized polynomial system of equations encodes the structure of the solutions. This group acts on the fiber of a projection from the incidence variety of parameters and solutions to the space of parameters. When this projection is decomposable, the Galois group is imprimitive, and we show that the structure can be exploited for computational improvements. We apply our method to problems in statistics, kinematics, and benchmark problems in computational algebra.

Waring decompositions and identifiability via Bertini and Macaulay2 softwares

Elena Angelini

Starting from our previous papers [AGMO] and [ABC], we prove the existence of a nonempty euclidean open subset whose elements are polynomial vectors with 4 components, in 3 variables, degrees, respectively, 2, 3, 3, 3 and rank 6, which are not identiable over C but are identiable over R. This result has been obtained via computer-aided procedures suitably adapted to investigate the number of Waring decompositions for general polynomial vectors over the elds of complex and real numbers. Furthermore, by means of the Hessian criterion ([COV]), we prove identiability over C for polynomial vectors in many cases of subgeneric rank.

On the postulation of lines and a fat line

Thomas Bauer, Sandra Di Rocco, David Schmitz, Tomasz Szemberg, Justyna Szpond

We show that a fat line of arbitrary multiplicity and an arbitrary number of general lines in P^3 impose independent conditions on forms of sufficiently high, effectively established, degree.

Regions of multistationarity in chemical reaction networks

Frédéric Bihan, Alicia Dickenstein, Magali Giaroli

Given a real sparse polynomial system, we present a general framework to find explicit coefficients for which the system has more than one positive solution, based on a recent article by Bihan and Spaenlehauer. We apply this approach to find explicit reaction rate constants and total conservation constants in biochemical reaction networks for which the associated dynamical system is multistationary.

Bordiga surface as critical locus for 3–view reconstruction in $\mathbb{P}^4$

Marina Bertolini, Roberto Notari, Cristina Turrini

In Computer Vision, images of suitable dynamic or segmented scenes are modeled as linear projections from $\mathbb{P}^4$ to $\mathbb{P}^2$. In this paper, we consider the 3–views case. At first, we declinate the Grassmann tensors introduced in [**?**] in our context, and we use them to compute the equations of the critical locus. Then, given the ideal that defines the critical locus, we prove that, in the general case, it defines a Bordiga surface, or a scheme in the same irreducible component of the associated Hilbert scheme. Furthermore, we prove that every Bordiga surface is actually the critical locus for the reconstruction for suitable projections.

On the existence of birational surjective parametrizations of affine surfaces

J. Caravantes, J.R. Sendra, D. Sevilla, C. Villarino

This paper deals with the problem of the existence of birational surjective parametrizations for affine rational surfaces. In particular, we prove that, if S is an affine complex surface whose projective closure is smooth, a necessary condition for S to admit a binational surjective parametrization from an open subset of the affine complex plane is that the infinity curve of S must contain at least one rational component.

We use this result to provide some examples of affine rational surfaces that do not admit surjective parametrization.

Bar code for monomial ideals

Michela Ceria

Aim of this paper is to count zerodimensional stable and strongly stable ideals in 2 and 3 variables, given their (constant) affine Hilbert polynomial. To do so, we introduce a bidimensional structure, called Bar Code which allows, a priori, to represent any finite set of terms M and, if M is an order ideal, to authomatically desume many properties of the corresponding monomial ideal I. For example, a Pommaret basis of I can be easily desumed. Then, we use it to give a connection between stable and strongly stable monomial ideals and integer partitions, providing formulas to count them.

Bounding the degrees of a minimal μ-basis for a rational surface parametrization.

Yairon Cid Ruiz

For an arbitrary rational surface parametrization $P(s,t) = (a_1(s,t), a_2(s,t), a_3(s,t), a_4(s,t)) \in F[s,t]$ over an infinite field F, we show the existence of a μ-basis with polynomials bounded in degree by $O(d^{33})$, where $d = \max(\deg(a_1), \deg(a_2), \deg(a_3), \deg(a_4))$. Our proof depends on obtaining a homogeneous ideal in $F[s,t,u]$ by homogenizing a_1, a_2, a_3, a_4. Making additional assumptions on this homogeneous ideal we can obtain lower bounds:

- If the projective dimension of this homogeneous ideal is one, then the bound for the μ-basis is d.
- If this homogeneous ideal has height three, then the bound for the -basis is in the order of $O(d^{22})$.
- When this homogeneous ideal is a general Artinian almost complete intersection, then the bound for the μ-basis is in the order of $O(d^{12})$.

On the resolution of fan algebras of principal ideals over a Noetherian ring

Teresa Cortadellas Benítez, Carlos Dandrea, Florian Enescu

We construct explicitly a resolution of a fan algebra of principal ideals over a Noetherian ring for the case when the fan is a proper rational cone in the plane. Under some mild conditions on the initial data, we show that this resolution is minimal.

Closed formula for univariate subresultants in multiple roots

Carlos Dandrea, Teresa Krick, Agnes Szanto, Marcelo Valdettaro

We generalize Sylvester single sums to multisets (sets with repeated elements) and show that these sums compute subresultants of two univariate polynomials as a function of their roots independently of their multiplicity structure. This is the first closed formula for subresultants in terms of roots that works for arbitrary polynomials, previous efforts only handled special cases. Our extension involves in some cases confluent Schur polynomials, and is obtained by using multivariate symmetric interpolation via an Exchange Lemma.

Computing the monodromy and pole order filtration on milnor fiber cohomology of plane curves

Alexandru Dimca, Gabriel Sticlaru

We describe an algorithm computing the monodromy and the pole order filtration on the Milnor fiber cohomology of any reduced projective plane curve C. The relation to the zero set of Bernstein-Sato polynomial of the defining homogeneous polynomial for C is also discussed. When C has some non weighted homogeneous singularities, then we have to assume that two conjectures hold in order to get some of our results. In all the examples computed so far this seems to be the case.

A Positivstellensatz for sums of nonnegative circuit polynomials

Mareike Dressler, Sadik Iliman, Timo De Wolff

Recently, the second and the third author developed sums of nonnegative circuit polynomials (SONC) as a new certificate of nonnegativity for real polynomials, which is independent of sums of squares. In this article we show that the SONC cone is full-dimensional in the cone of non-negative polynomials. We establish a Positivstellensatz which guarantees that every polynomial which is positive on a given compact, semi-algebraic set can be represented by the constraints of the set and SONC polynomials. Based on this Positivstellensatz we provide a hierarchy of lower bounds converging against the minimum of a polynomial on a given compact set K. Moreover, we show that these new bounds can be computed efficiently via interior point methods using results about relative entropy functions. In summary, our results establish SONCs as a compelling alternative to sums of squares as nonnegativity certificates both in theory and practice.

On the number of vertices of generic spectrahedra

Hamza Fawzi, Mohab Safey El Din

A vertex of a convex body is an extreme point where the normal cone is full-dimensional. Vertices play the role of singularities on boundaries of convex bodies. It is known that any convex body has at most a countable number of vertices. In this paper we consider the following question: how many vertices can a spectrahedron defined using a linear matrix inequality of size $m \times m$ have? Using standard arguments from algebraic geometry, we give an upper bound of $2^{O(m^2)}$ on the number of such vertices under certain genericity assumptions. In certain regimes the upper bound simplifies to $2^{O(m)}$ which is attained by some specific spectrahedra. It is open whether there exist spectrahedra with $2^{\Omega(m^2)}$ vertices.

On semiring complexity of Schur polynomials

Sergey Fomin, Dima Grigoriev, Dorian Nogneng, Éric Schost

Semiring complexity is the version of arithmetic circuit complexity that allows only two operations: addition and multiplication. We show that semiring complexity of a Schur polynomial $s_\lambda(x_1, \ldots, x_k)$ labeled by a partition $\lambda == (\lambda_1 \geq \lambda_2 \geq \ldots)$ is bounded by $O(\log(\lambda_1))$ provided the number of variables k is fixed.

Counting projections of rational curves

Matteo Gallet, Josef Schicho

We consider the following problem: we are given two non-degenerate rational curves of the same degree lying in two (possibly different) projective spaces, and we ask whether there exists a third rational curve of the same degree in another projective space, together with two linear projections mapping it to each of the initial two curves. When the dimensions of these three projective spaces and the degree of the curves satisfy some linear equation, one expects that the number of such projections should be finite. We prove that this is indeed the case, and we provide a formula for such number.

Algorithms for tight spans and tropical linear spaces

Simon Hampe, Michael Joswig, Benjamin Schröter

We describe a new method for computing tropical linear spaces and more general duals of polyhedral subdivisions. It is based on Ganter's algorithm (1984) for finite closure systems.

Interlacing Ehrhart polynomials arising from graphs

Akihiro Higashitani, Mario Kummer, Mateusz Michalek

It was observed by Bump et al. that Ehrhart polynomials in a special family exhibit properties similar to the Riemann function. The construction was generalized by Matsui et al. to a larger family of reflexive polytopes coming from graphs. We prove several conjectures confirming when such polynomials have zeros on a certain line in the complex plane. Our main new method is to prove a stronger property called interlacing.

Imaginary projections of polynomials

Thorsten Jörgens, Thorsten Theobald, Timo De Wolff

We introduce the imaginary projection $\mathcal{I}(f)$ of a multivariate polynomial $f \in \mathbb{C}[\mathbf{z}]$ as the projection of the variety of f onto its imaginary part. Since a polynomial f is stable if and only if $\mathcal{I}(f) \cap \mathbb{R}^n_{>0} = \varnothing$, the notion offers a novel geometric view underlying stability questions of polynomials.

We show that the connected components of the complement of the imaginary projections are convex, thus opening a central connection to the theory of amoebas and coamoebas. Building upon this, we establish structural properties of the components of the complement, such as lower bounds on their maximal number, prove a complete classification of the imaginary projections of quadratic polynomials and characterize the limit directions for polynomials of arbitrary degree.

On the toric ideals of matroids of fixed rank

Michal Lason

In 1980 White conjectured that the toric ideal associated to a matroid is generated by quadratic binomials corresponding to symmetric exchanges. Herzog and Hibi go even further they ask if the toric ideal of a matroid possesses a Grobner basis of degree 2. We study these problems for a class of matroids of fixed rank, and obtain several finiteness results.

We prove Whites conjecture for high degrees. That is, we prove that for all matroids of fixed rank r, homogeneous parts of degree at least $c(r)$ of the corresponding toric ideals are generated by quadratic binomials corresponding to symmetric exchanges. This extends our previous result (with Mateusz Michalek) confirming the conjecture up to saturation.

We also prove that for the class of matroids of fixed rank, there exists a common upper bound on the degree of a Grobner basis. Namely, we prove that the toric ideal of a matroid of rank r possesses a Grobner basis of degree at most $2(r+3)!$.

First result (Whites conjecture for high degrees) has been already submitted to a journal. Second result (about Grobner bases) is an ongoing project.

Beyond polyhedral homotopies

Anton Leykin, Josephine Yu

Given an affine variety we consider the problem of finding isolated solutions, on the given variety, of a polynomial system that is sparse with respect to a non-monomial basis. We construct a method to solve this problem using tropical geometry and homotopy continuation machinery. As a part of our algorithm we also get the mixed volume of Newton–Okounkov bodies without having to compute the bodies. This approach generalizes the polyhedral homotopies by Huber and Sturmfels.

Surfaces containing two circles through a general point in higher dimensions

Niels Lubbes

We investigate real surfaces in projective spheres that contain at least two circles through a general point. Automorphisms of the projective sphere are called Moebius transformations. In order to classify such surfaces, we consider several Moebius invariants such as singular locus, Picard lattice, toric structure and surface automorphisms that are Moebius transformations. In particular, we classify surfaces whose Moebius automorphisms act transitively on their real points.

An arithmetic Bernstein-Kusnirenko inequality

César Martiínez, Martín Sombra

We present an upper bound for the height of the isolated zeros in the torus of a system of Laurent polynomials over an adelic field satisfying the product formula. This upper bound is expresed in terms of the mixed integrals of the local roof functions associated to the chosen height function and to the system of Laurent polynomials. We also show that this bound is close to optimal in some families of examples. This result is an arithmetic analogue of the classical Bernstein-Kusnirenko theorem. Its proof is based on arithmetic intersection theory on toric varieties.

Validity proof of Lazard' s method for CAD construction

Scott Mccallum, Adam Parusiński, Laurentiu Paunescu

In 1994 Lazard proposed an improved method for cylindrical algebraic decomposition (CAD). The method comprised a simplified projection operation together with a generalized cell lifting (that is, stack construction) technique. For the proof of the methods validity Lazard introduced a new notion of valuation of a multivariate polynomial at a point. However a gap in one of the key supporting results for his proof was subsequently noticed. In the present paper we provide a complete validity proof of Lazards method. Our proof is based on the classical parametrized version of Puiseuxs theorem and basic properties of Lazards valuation. This result is significant because Lazards method can be applied to any finite family of polynomials, without any assumption on the system of coordinates. It therefore has wider applicability and may be more efficient than other projection and lifting schemes for CAD.

Finite phylogenetic complexity and combinatorics of tables

Mateusz Michalek, Emanuele Ventura

In algebraic statistics, Jukes-Cantor and Kimura models are of great importance. Sturmfels and Sullivant generalized these models associating to any finite abelian group G a family of toric varieties $X(G, K_{1,n})$. We investigate the generators of their ideals. We show that for any finite abelian group G there exists a constant ϕ, depending only on G, such that the ideals of $X(G, K_{1,n})$ are generated in degree at most ϕ.

Symbolic computation in hyperbolic programming

Simone Naldi, Daniel Plaumann

Hyperbolic polynomials can be thought of as generalized characteristic polynomials of real symmetric matrices. Hyperbolic programming is the problem of computing the infimum of a linear function when restricted to the hyperbolicity cone of such a polynomial, a generalization of semidefinite programming. We propose an approach based on symbolic computation, relying on the multiplicity structure of the algebraic boundary of the cone, without the assumption of determinantal representability. This allows us to design exact algorithms able to certify the multiplicity of the solution and the optimal value of the linear function.

Border ranks of monomials

Luke Oeding

Young flattenings, introduced by Landsberg and Ottaviani, give determinantal equations for secant varieties and provide lower bounds for border ranks of tensors. We find special monomial-optimal Young flattenings that provide the best possible lower bound for all monomials up to degree 6. For degree 7 and higher these flattenings no longer suffice for all monomials. To overcome this problem we introduce partial Young flattenings and use them to give a lower bound on the border rank of monomials which agrees with Landsberg and Teitler's upper bound.

Optimization approaches to quadrature

Cordian Riener, Markus Schweighofer

Let d be a positive integer and μ be a positive Borel measure on the real plane possessing moments up to degree $2d1$. A quadrature rule for μ is a finite set of points $\{x_1, \ldots, x_N\}$, together with associated non-negative weights such that integration with respect to μ of any degree $2d-1$ polynomial f equals the weighted point evaluation on these points, i.e., we have

$$\int f d\mu = \sum_{i=1}^{N} \lambda_i f(x_i).$$

In this talk, we present an optimization approach to the question how many nodes are necessary for such a quadrature rule. We show that in the case when the support of μ is contained in an algebraic curve of degree k that there exists a quadrature rule for μ with at most dk many nodes all placed on the curve (and positive weights) that is exact on all polynomials of degree at most $2d1$. This generalizes both Gauss and (the odd degree case of) Szegő quadrature where the curve is a line and a circle, respectively, to arbitrary plane algebraic curves. We use this result to show that, without any hypothesis on the support of μ, there is always a cubature rule for μ with at most $3/2d(d1) + 1$ many nodes. In both results, we show that the quadrature rule can be chosen such that its value on a certain positive definite form of degree 2d is minimized. Finally, we also characterize the unique Gaussian quadrature rule on the line as the one that minimizes a "penalty function" - for example the maximum distance of a node to the origin. The tools we develop should prove useful for obtaining similar results in higher-dimensional cases although at the present stage we can present only partial results in that direction.

A-discriminants for complex exponents and counting real isotopy types

J. Maurice Rojas, Korben Rusek

We extend the notion of A-discriminant, and Kapranov's parametrization of A-discriminant varieties, to complex exponents. As an application, we focus on the special case where A is a set of $n + 3$ points in real n-space with non-defective Gale dual, g is a real n-variate exponential sum with spectrum A and sign vector s, and prove the following: For fixed A and s, the number of possible isotopy types for the real zero set of g is $O(n^2)$. We are unaware of any earlier such upper bound, except for an $O(n^6)$ bound when all the points of A lie in $\mathbb{Z}^n$.

On the unirationality of Hurwitz ppaces

Frank-Olaf Schreyer, Fabio Tanturri

A method to prove the unirationality of a moduli space of curves or other interesting moduli spaces is to explicitly produce a dominant unirational family of projective models. In this talk I will focus on the unirationality of the Hurwitz spaces $H_{g,d}$ parameterizing d-sheeted branched simple covers of the projective line by smooth curves of genus g. After a brief summary about the known results, I will present the construction of two unirational families of curves dominating $H_{12,8}$ and $H_{13,7}$, obtained respectively via matrix factorizations and via liaison techniques.

Optimal curve parametrization and an application to algebraic ODEs

N. Thieu Vo, Georg Grasegger, Franz Winkler

We consider first-order algebraic ordinary differential equations (AODEs) and study their rational general solutions. A rational general solution of a first-order AODE contains an arbitrary constant. In case the constant appears rationally, we call the solution strong. We present an algorithm for deciding the existence of a strong rational general solution of a first-order AODE, and in the positive case, compute such a solution. Our method is based on optimal parametrizations of algebraic curves over the field of rational functions.

Tropical geometry of toric stacks

Martin Ulirsch

The goal of our project is to develop the tropical geometry of toric stacks expanding on the already existing theory for toric varieties (see [Kaj08] and [Pay09]). In this presentation I intend to give an introduction to the current status of our theory with an emphasis on some anticipated applications as well as to the combinatorial tools that are useful for explicit descriptions of our tropical toric stacks. The main example will be a stack-theoretic generalization of the well-known Losev-Manin moduli spaces [LM00] using the theory of twisted curves, as developed in [AV02] and [Ols07], so called twisted Losev-Manin spaces.

3 Computation presentations

On the discrete logarithm problem for prime-field elliptic curves

Alessandro Amadori, Michela Ceria, Federico Pintore, Massimiliano Sala

In recent years many papers have appeared investigating the classical discrete logarithm problem for elliptic curves, exploiting the multivariate polynomial approach with the use of the celebrated summation polynomials, introduced by Semaev in 2004. However, with a notable exception by Petit et al., all numerous specialized papers have investigated only the composite-field case, leaving apart the laborious prime-field case. In this extended abstract we propose a variation of Semaevs original approach for the prime-field case. Our proposal outperforms both the original Semaev's method and Petit's specialized algorithm. We reach our improvement by reducing the necessary Groebner basis computations to only one basis calculation.

Extraction of tori from minimal point sets

Laurent Busé, André Galligo

In this extended abstract, a new algebraic method for extracting tori from a minimal point set, made of two oriented points and a simple point, is proposed. We provide a degree bound on the number of such tori; this bound is reached on examples, even when we restrict to smooth tori. Our method is based on pre-computed closed formulae that provide an efficient code, well suited to numerical computations with approximate input data.

Computing resolutions of equivariant modules

Sebastian Posur

We present a method for the computation of projective and injective resolutions of finitely generated equivariant modules over a finite dimensional equivariant algebra. For this, we will use the concept of an internal module from category theory. The case of the graded exterior algebra will then give us a method for the computation of equivariant sheaf cohomology on projective space via the so called Bernstein-Gel'fand-Gel'fand correspondence. All the presented ideas are realized within the Cap project, a collection of software packages for category theory implemented in the computer algebra system GAP.

Computing tropical points and tropical links using intersections with affine hyperplanes

Yue Ren, Tommy Hofmann

We present a fast algorithm for computing zero-dimensional tropical varieties using triangular decomposition and Newton polygons. From it, we derive algorithms for computing tropical points and tropical links using intersections with affine hyperplanes. We present an implementation within Singular's framework for computing tropical varieties.

4 Posters

Families of stable locally free sheaves on projective spaces

Charles Almeida, Marcos Jardim, Alexander Tikhomirov, Sergey Tikhomirov

We present a family of monads whose cohomology are stable rank 2 locally free sheaves on P3. Then we study the irreducibility and smoothness together with a geometrical description of some these fam- ilies. As an application, we proved that the moduli space of stable rank 2 locally free sheaves with rst Chern equals to 0 and second Chern class equals to 5 has exactly 3 irreducible components.

A new model of avalanche transceiver

Alberto Alzati, Cristina Turrini, Giuseppe Rotondo, Gianfranco Venturino, Iuri Frosio

An avalanche transceiver, commonly called ARTVA, is a device allowing the research of peoples buried under avalanches. The transceiver equipment currently in use produce a magnetic field and the researchers follow the tangent directions to the flux curves of the field until they arrive near the victims. In this poster we describe a new method of research, based on the geometrical properties of the above curves, allowing the researchers to walk towards the victims along straight lines. This method (patent 102016000087706 filed on August 29th 2016, Uff. Italiano Brevetti e Marchi) could be implemented in the existing ARTVAs with minor changes.

Orthogonal instanton bundles on P^3

Aline Andrade, Simone Marchesi, Rosa Maria Miró-Roig

In this work we provide a bijection between equivalence classes of orthogonal instanton bundles on P^3 and symmetric forms. Using such correspondence, we prove the non-existence of orthogonal instanton bundles on P^3, with second Chern class equals to one or two, and using Macaulay2 we also provide explicit examples of orthogonal instanton bundles on P^3 of second Chern classes greater or equal than 3.

Noether resolutions in dimension 2

Isabel Bermejo, Eva Garcia-Llorente, Ignacio Garcia Marco and Marcel Morales

Let $R := K[x_1, \dots, x_n]$ be a polynomial ring over an infinite field K, and let $I \subset R$ be a homogeneous ideal with respect to a weight vector $\omega = (\omega_1, \dots, \omega_n) \in (\mathbb{Z}^+)^n$ such that $\dim(R/I) = d$. In this work we study the minimal graded free resolution of R/I as A-module, that we call the Noether resolution of R/I, whenever $A := K[x_{n-d+1}, \dots, x_n]$ is a Noether normalization of R/I. When $d = 2$ and I is saturated, we give an algorithm for obtaining this resolution that involves the computation of a minimal Gröbner basis of I with respect to the weighted degree reverse lexicographic order. In the particular case when R/I is a 2-dimensional semigroup ring, we also describe the multigraded version of this resolution in terms of the underlying semigroup. Whenever we have the Noether resolution of R/I or its multigraded version, we obtain formulas for the corresponding Hilbert series of R/I, and when I is homogeneous, we obtain a formula for the Castelnuovo-Mumford regularity of R/I.

An implicitization algorithm based on Chow Forms for codimension larger than one

Ioannis Z. Emiris, Clément Laroche, Christos Konaxis

We develop a randomized algorithm based on the theory of Chow Forms and the geometry of conical surfaces to compute implicit equations of algebraic varieties of codimension larger than one. In particular, given parametric equations of a space curve, we are able to compute 3 pencils of lines containing this curve and describing it as a set-theoretical intersection; the surfaces' degree equals the degree of the parametric equations. In general, the output equations are of near-optimal degree, are correct even in presence of base points or self-intersections, while the complexity of the algorithm is that of a resultant computation per implicit equation. We have implemented the method in Maple, both for space curves and the higher dimensional setting, and have compared it with several other methods:

- Maple's native method is slower, and returns more equations than necessary for defining the variety,
- A matrix representation algorithm is a bit slower in low degree, faster in higher degree (¿7), but does not yield closed formulas,
- An implicitization algorithm using a Grbner basis computation is faster than ours, in particular in higher degree, returns usually more equations than our algorithm, but of lesser degree.

Primary decomposition of certain determinantal ideals

Indranath Sengupta, Joydip Saha, Gaurab Tripathi

Our aim is to compute primary decomposition of ideals of the form $I_1(XY)$. We use the technique of Gröbner bases.

Semi-algebraic decomposition of real binary forms

Maria-Angeles Zurro, Macarena Ansola, Antonio Diaz-Cano

The classical Waring Problem over the ring of polynomials in two real variables states that any form of degree d can be written as a nite sum of d-th powers of linear forms. Based on the Sylvester's Algorithm adapted to the real eld, we give a Waring decomposition with minimal length, say r. Furthermore, our algorithm allows us to determine semialgebraic conditions to characterize the set of real binary forms of real rank r. The space of real binary forms of degree d has a semialgebraic decomposition by means of its real rank. Those forms of real rank r, $W(r)$, are presented as a union of semialgebraic strata with $\dim(W(r)) = d + 1$, for $[d/2] + 1 \leq r \leq d$ (since these ranks are typical).

Organizing committee:

Ada Boralevi, Politecnico di Torino

Laurent Busé, Inria Sophia Antipolis Méditerranée

Agnes Cortell, Inria Sophia Antipolis Méditerranée

André Galligo, Université de Nice

Angélique Guitard, Université de Nice

Evelyne Hubert, Inria Sophia Antipolis Méditerranée

Bernard Mourrain, Inria Sophia Antipolis Méditerranée

Sponsors

An event organized by

Member of UNIVERSITÉ CÔTE D'AZUR